UNCLASSIFIED

AD NUMBER AD406191 NEW LIMITATION CHANGE TO Approved for public release, distribution unlimited **FROM** Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 15 MAY 1963. Other requests shall be referred to Air Force Rome Air Development Center, Research and Technology Division, Griffiss AFB, NY. **AUTHORITY** RADC ltr, 3 Jun 1965

UNCLASSIFIED Anc 101

AD 406 191

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

406191

FINAL REPORT SYSTEM RELIABILITY PREDICTION BY FUNCTION

406 191

Federal Electric Corporation
Equipment and Systems Evaluation Branch
Engineering and Support Services Division
Industrial Park
Paramus, New Jersey

Contract AF30(602)2687

Prepared for

Rome Air Development Center
Research and Technology Division
Air Force Systems Command
United States Air Force

Griffiss Air Force Base New York

PUBLICATION REVIEW

This report has been reviewed and is approved.

Approved:

DAVID F. BARBER

Chief, Applied Research Laboratory

Directorate of Engineering

Approved:

AM BETHKE

Director of Engineering

FOR THE COMMANDER!

RVING-J. GABELMAN

Director of Advanced Studies

FINAL REPORT

SYSTEM
RELIABILITY PREDICTION
BY FUNCTION

Federal Electric Corporation
Equipment and Systems Evaluation Branch
Engineering and Support Services Division
Industrial Park
Paramus, New Jersey

Contract AF30(602)2687

Project Number: 5519 Task Number: 551902

Prepared

for

Rome Air Development Center Research and Technology Division Air Force Systems Command United States Air Force

> Griffiss Air Force Base New York

FOREWORD

"This final technical report describes the results of a study conducted by Federal Electric Corporation to develop a technique for predicting the reliability of a system or sub-system during the early planning stages of design. In the early planning stages the designer knows or can accurately estimate certain functional parameters of an equipment. Such functional parameters include the range or power output of a transmitter, the bandwidth or sensitivity of a receiver and the speed or capacity of a data processing equipment. The System Reliability Prediction by Function Technique is predicated on these functional parameters and on the number of Active Element Groups associated with each function.

It is the opinion of the Rome Air Development Center that the contractor has successfully completed the objective of the study for Radar and Communication-type equipments. A Prediction by Function Technique for data processing equipments was not developed due to the insufficiency of operational data, despite the wholehearted attempts by the contractor to obtain the necessary data.

The Rome Air Development Center is cognitive of the need for expansion of the technique to include equipment characteristics not covered in the original effort and equipment using other than conventional parts. A program to further refine and verify the technique, thus establishing and maintaining a high degree of confidence in its results, should also be initiated."

ABSTRACT

a new completions of the second of the secon

The purpose of this study contract was to develop techniques for predicting the reliability of electronic equipments and systems during the early planning stages. The techniques were to be based on correlation factors which would relate system, equipment, or sub-equipment functions (radar transmitters for instance) by specific characteristics (such as, peak power) to reliability levels achievable within the existing state-of-the-art. The study was conducted under contract number AF30(602)2687 which commenced in February 1962 and continued for a period of thirteen months.

The correlation studies performed were categorized into three areas; radar, ground communication and data processing. A number of applicable radar and ground communication systems were selected for study and definite correlation was found between functional characteristics and actual field MTBF. The results of these correlation studies led to the development of prediction techniques for both the radar and ground communication categories.

Verification of these techniques was accomplished by applying them to other systems within the appropriate category and comparing the resultant predictions to those calculated from actual field data. The results proved very satisfactory thus leading to the conclusion that the prediction techniques were sound.

Difficulty was encountered however in acquiring appropriate field operational and failure data needed for correlating the reliability of data processing systems to functional characteristics. Therefore, no firm technique was developed for this category. However, the basic approaches

to the problem of isolating particular functional levels which can be correlated with the reliability of data processing equipments are discussed herein.

see entry

TABLE OF CONTENTS

The Company of the Company

C

Section		Pag
1	INTRODUCTION Objective Scope Background	1 1 3
2	TECHNICAL DISCUSSION	5 5 6
	Of Radar	8
	Reliability Prediction of the Receiver Function	14
	Reliability Prediction of the Indicator Function	
	Radar SystemsVerification of Reliability Prediction By Function Technique for Radar	24
	Radar "A" Verification	30
	Communication Systems	
	Reliability Prediction of the Transmitter, Exciter and Receiver Function Reliability Prediction of the Multiplex	34
	Function	41 43
	Verification of Reliability Prediction By Function Technique for Ground Commu-	
	nication SystemsAN/FRC-47(V) Verification	44
	MTBF's for AN/TRC-29 Microwave System Data Processing Systems	
3	CONCLUSIONS AND RECOMMENDATIONS	51

LIST OF ILLUSTRATIONS

FIGURE		PAGE
I	Normalized MTBF's (HRS) vs. Transmitter Peak Power (KW)	13
2	Nomograph for Determining Radar Transmitter Failure Rate	15
3	Receiver Noise Safety Margin vs. Failures Per Active Element	19
4	AN/FRC-45(V) IKW, Simplified Diagram	35
5	AN/FRC-39A(V) IKW, Simplified Diagram	35
6	Relationship Between C/N Ratio and MTBF For Combined Transmitter, Exciter and Receiver Functions of Ground Communication Systems.	40

LIST OF TABLES

TABLE		PAGE
1	Actual MTBF's of Radar Functions	. 10
2	Normalized Field MTBF's of Radar Functions	. 11
3	Radar Transmitter Peak Power and Normalized MTBF	. 12
4	Receiver Noise Figure/Signal-To-Noise Ratios and Normalized MTBF's	. 17
5	Receiver Noise Safety Margins and Normalized Failure Rates	. 18
6	Comparison of Radar "A" Predicted and Actual MTBF's	. 28
7	Tropospheric Scatter Communication Systems Analyzed	. 32
8	Actual MTBF's of Tropo Functions	. 33
9	Ground Communication Systems Calculated C/N Ratios and MTBF's	. 37
10	System C/N Ratio and MTBF Rank Order Correlation	. 37
[]	Comparison of AN/FRC-47(V) Predicted and Actual MTBF's	. 46
12	Comparison of AN/TRC-29 Predicted and Actual MTBF's	. 47



LIST OF APPENDICES

1. On the state of the state of

Appendix I - Summary of Procedures for Reliability Prediction of Radar and Ground Communication Systems.

į

et a reas afficience of a

INTRODUCTION

<u>Objective</u>: The purpose of this study was the development of electronic system oriented prediction techniques by correlating significant functional characteristics to operational Mean-Time-Between-Failures (MTBF's) so that system reliability can be planned and allocated during the early design stages.

It is intended that these techniques be used by system design engineers to estimate the reliability of a system during the early phases of the design cycle when no firm data is available concerning the ultimate configuration to be employed or the total part compliment of the system. The techniques provide a means of relating the expected reliability of one or more proposed design concepts to that of an alternate choice or choices so that knowledgeable quantitative trade-off studies can be conducted and decisions reached that will assist in determining if the proposed system design meets the required goals.

Scope: These correlation studies were applied to broad system areas and are applicable to different equipment functional levels, ranging as specified from command to data acquisition, to receivers, and transmitters. The procedures are not intended to replace the conventional methods of reliability prediction such as stress analysis, as is contained in the RADC Reliability Notebook TR-58-III. Instead, they are to be used to augment them in the early pre-design planning stages at a time when specific part selection has not been defined.

evolved in this study. Using these techniques, reliability prediction is accomplished for radar and ground communication categories by either extracting proper quantities from correlation curves or by suitable caiculations taken from correlation formulae developed during this study. Appendix I, "Summary of Procedures for Reliability Prediction of Radar and Ground Communication Systems" contains the step by step procedures for predicting the reliability of radar and ground communication systems. These procedures are based on the techniques described in Section 2, Technical Discussion.

Difficulty was encountered in acquiring appropriate field operational and failure data needed for correlation of electronic data processing systems.

Therefore no firm prediction technique was developed for this category.

However a discussion is presented which concerns the basic approaches taken and the results of the work accomplished to date in this area.

Background

. . . .

1

Reliability prediction, as a tool applicable to the first stages of system design, has changed in character from early days of piece-part stress analysis. It is now necessary to use specific and tailored approaches particularly when dealing with modern complexes such as weapon control and navigational systems. Also, improvement in electronic piece-part quality of construction during the last five years has made conventional stress analyses more routine and restricted to the indication of pure catastrophic tendencies at the piece-part level. Simultaneously, equipment complexity is compounded by computer oriented control and measurement devices making even more necessary the sophistication of system planning and analysis.

TECHNICAL DISCUSSION

General

This portion of the report concerns the detailed description of the development of techniques for predicting the reliability of electronic equipments and systems during the early (pre-design) system planning stages. Specifically, it concerns the treatment of Radar, Ground Communication and Data Processing categories.

The approach to be followed concerned the correlation of pertinent electrical parameters or characteristics for the functional requirements (e.g., receiver) of the system design with actual field MTBF data. For each category design variables inherent to that category were investigated for correlation.

For radar systems all the parameters included in the radar equation as well as bandwidth and frequency were investigated both independently and combined. It will be seen that a firm correlation was found between radar transmitter peak power and actual MTBF, however, no single receiver parameter exhibited consistent correlation. A figure of merit called the Noise Safety Margin, based on noise figure or signal-to-noise ratio, was developed and good correlation was indicated.

Similar approaches were followed for ground communication systems. Correlation of actual MTBF for the combined function of transmitter, exciter and receiver with the carrier-to-noise ratio were observed.

As discussed previously, difficulty was encountered in obtaining a significant amount of data for data processing equipments thereby precluding the development of a prediction technique for this category.

The following paragraphs discuss the development and application of the prediction techniques in detail. It should be noted that a negative correlation of actual MTBF with a system parameter does not necessarily mean that no correlation is possible, it does mean, however, that none was apparent from the data analyzed during this study.

Radar

The following presents a technique for predicting the reliability of pursed radar systems. The fundamental objective of this prediction technique is to provide a designer with a method of estimating Mean-Time-Between-Failure (MTBF) which will be representative of its performance when operating under field environmental conditions. The technique provides the means of estimating radar system MTBF as well as individual MTBF's for the innerent functional requirements of transmitter, receiver and indicator. It was developed through correlation of actual field MTBF data with pertinent electrical parameters or characteristics for the functional requirements of radar system design.

Basic Design Parameters of Radar

The purpose of radar is to present information on the position of targets within the volume of space it surveys. In discussing the performance of a radar system, one usually refers to its range performance. The radar range performance or free-space maximum detection range is the range beyond which a target cannot be detected because the echo signals are obscured by noise.

To estimate the range performance of a radar, a designer usually calculates its free-space maximum detection range using the inverse-square law. The inverse-square law, which governs the intensity of radiation from a point source, is used to determine the range dependence of the fraction of the

Thus we see two significant factors emerging in modern reliability:

- 1) The necessity of using a system approach since the majority of equipments eventually become integrated within some electronic complex.
- 2) The importance of functionally oriented reliability analysis during early hardware design stages and subsequently throughout all other phases of a project.

It is believed that this study has encompassed both these factors and that the techniques developed will provide a means of relating the reliability of proposed equipment and system design concepts to that of alternate choice or choices. In this way knowledgeable quantitative trade-off studies can be conducted and decisions reached that will ensure that proposed systems meet their required goals.

evolved in this study. Using these techniques, reliability prediction is accomplished for radar and ground communication categories by either extracting proper quantities from correlation curves or by suitable caiculations taken from correlation formulae developed during this study. Appendix I, "Summary of Procedures for Reliability Prediction of Radar and Ground Communication Systems" contains the step by step procedures for predicting the reliability of radar and ground communication systems. These procedures are based on the techniques described in Section 2, Technical Discussion.

Difficulty was encountered in acquiring appropriate field operational and failure data needed for correlation of electronic data processing systems. Therefore no firm prediction technique was developed for this category. However, a discussion is presented which concerns the basic approaches taken and the results of the work accomplished to date in this area.

Background

Reliability prediction, as a tool applicable to the first stages of system design, has changed in character from early days of piece-part stress analysis. It is now necessary to use specific and tailored approaches particularly when dealing with modern complexes such as weapon control and navigational systems. Also, improvement in electronic piece-part quality of construction during the last five years has made conventional stress analyses more routine and restricted to the indication of pure catastrophic tendencies at the piece-part level. Simultaneously, equipment complexity is compounded by computer oriented control and measurement devices making even more necessary the sophistication of system planning and analysis.

*****.,

total transmitted energy that falls on a target, considering the target as a point source of radiation.

The inverse-square law is also used to determine the range dependence upon the amount of echo energy re-radiated from the target to the receiver. As a result of the radiated and re-radiated signal the echo energy received from a target varies with the inverse fourth power of the range from the radar to the target. Thus, the over-all ability of a radar to detect targets therefore depends upon the ratio of its transmitted-pulse energy to detectable echo-energy. For a given radar this ratio may vary widely because of propagation factors and conditions under which the transmitter and receiver are operated.

A typical equation that may be used by a designer to calculate the maximum free-space detection range of a radar during its early planning stages, is as follows:

$$R_{\rm m} = 0.1146$$

$$P_{\rm p} \tau \sigma \lambda^2 G^2 L_1 L_2^2/VN \qquad (1)$$

where:

 R_m = maximum free space range in statute miles

 P_D = peak power in kilowatts

 γ = pulse width in microseconds

 σ = effective target area in square feet

 λ = carrier wavelength in centimeters

G = antenna gain

L, = line losses

[&]quot;Reference Data for Radio Engineers", Fourth Edition, International Telephone and Telegraph Corporation, July 1957, Chapter 27, Page 808.

L = transmission losses

V = indicator visibility factor

N = receiver noise tigure

The significant factors listed above were statistically and analytically considered for correlative indications in regard to the reliability of radar systems. Investigation of pulse width, carrier frequency, transmission losses and antenna constants (target area, gain, line losses) gave no indication of firm relationship to reliability when correlated with the actual field MTBF data available.

As indicated above, definite correlation was obtained between actual MTBF data and typical radar electrical performance characteristics of transmitter peak power and receiver sensitivity. Early investigations likewise indicated some correlation of field data with the visibility factor of indicators.

Based on the correlations discussed above and on practical considerations, the radar systems analyzed were reduced to three basic functional levels: Transmitter, Receiver and Indicator. This functional grouping was used since the electrical principles of operation (transmit, receive, indicate) are essentially the same for all radars, even though radar systems now in existence vary greatly as to detail in design and construction. Integral studies were therefore conducted upon these three inherent functional levels. Radar system correlation studies accordingly were based on the composite effects derived from the functional characteristics of all three of the above levels.

Approach to Reliability Prediction of Radar

Correlation studies were conducted using field operational and failure data accumulated by FEC during the operation and maintenance of DEWLine, Senorita,

Pacific Missile Range and other large-scale projects. The radars included in the development of the prediction techniques discussed below were based on an analysis of four (4) basic radar systems, namely:

- I) AN/FPS-19 L-Band Type Radar
- 2) AN/FPS-16 Amplitude Comparison Radar
- 3) ARSR-IA Air Route Surveillance Radar
- 4) AN/GPX-26 IFF Recognition (Radar)

These four radars comprised a total field operational population of one hundred and twenty (120).

The field data analyzed covered a one year period of operation from I January 1961 to 31 December 1961. In analyzing the data a failure was defined as a detected cessation of ability to perform a specific function within previously established limits in the areas of interest. It is a malfunction which is beyond adjustment by the operator by means of controls normally accessible to him during the routine operation of the device. Table I below, "Actual MTBF's of Radar Functions", shows the radar type, functional level and respective actual field MTBF's calculated from operational data.

An examination of the data presented in Table I shows that the actual field MTBF for the same radar function varies considerably between radar types. For example, the AN/FPS-19 Transmitter MTBF is 1245 hours, the ARSR-1A Transmitter MTBF is 546 hours, and the AN/FPS-16 Transmitter MTBF is 370 hours. This spread in MTBF range would normally be attributed to a difference in transmitter part complexity. Investigation of the part complexities of these three transmitters, however, reveals that there is no substantial difference between these two equipments and the AN/FPS-16. It is apparent then, that in addition to the part complexity of the transmitter, there are other

parameters (e.g., peak power, etc.) which seriously affect its field MTBF.

TABLE I

ACTUAL MTBF'S OF RADAR FUNCTIONS

RADAR TYPE	FUNCTION	ACTUAL FIELD MTBF (HRS.)
AN/FPS-19	Transmitter	1245
,	Receiver	658
	PPI Indicator	3185
	A Indicator	10369
ARSR-1A	Transmitter	546
	Receiver	141
	PPI Indicator	2339
AN/FPS-16	Transmitter	370
,	Receiver	123
	A/R Indicator	4814
AN/GPX-26	Transmitter	4342
·	Receiver	6598
	Indicator	*
*Since the AN/GPX-26 is	s IFF, i† does not use ar	n Indicator.

In order to determine if a parameter such as the peak power of transmitters can be correlated with their field MTBF's, the part complexity difference between transmitters had to be adjusted. To accomplish this, a "normalized" field MTBF was used. The normalized field MTBF was calculated by multiplying the actual field MTBF of a function by the number of active element groups within the function. An active element group is defined as a "vacuum tube and its associated parts". The reason for multiplying by the number of active element groups is that the number of parts used per tube does not vary substantially between these transmitters, that is, the parts

used per tube in these transmitters range from 12 to 15. Through this method, the effect of the difference in the number of parts used per function type is held constant and correlation analysis of its electrical parameters with the "normalized" field MTBF's is made possible. The normalized field MTBF's that were calculated for the radar equipments by function are shown in Table 2, "Normalized Field MTBF's of Radar Functions".

TABLE 2

NORMALIZED FIELD MTBF'S OF RADAR FUNCTIONS

RADAR <u>TYPE</u>	FUNCTION	NORMALIZED F MTBF (HRS)
AN/FPS-19	Transmitter	17430
,,,,,,,,,	Receiver	32241
	PP1 Indicator	66885
	A Indicator	9 3 2.83
ARSR-IA	Transmitter	6549
	Receiver	17485
	PPI Indicator	201200
AN/FPS-16	Transmitter	11470
	Receiver	19920
	A/R indicator	81833
AN/GPX-26	Transmitter	52104
•	Receiver	79176
	Indicator	*

Reliability Prediction of the Transmitter Function

The functional characteristic of peak power exhibited a consistent correlation with actual MTBF's of the transmitter function for the radars studied. Consistent correlation is defined here as, "the property where an upward or downward trend is exhibited with no reversals". Table 3, "Radar Transmitter Peak Power and Normalized MTBF", which lists transmitter type, peak

power and normalized field MTBF's illustrates this trend.

TABLE 3

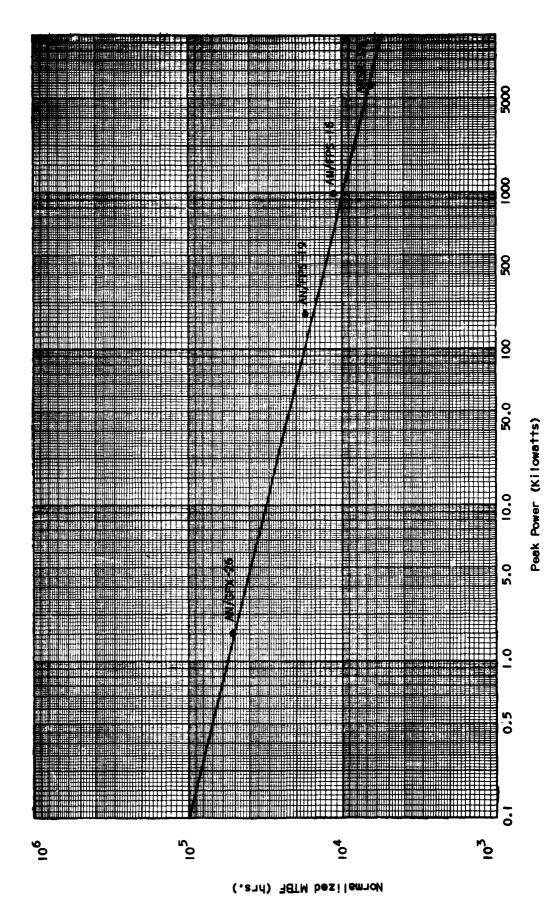
RADAR TRANSMITTER PEAK POWER AND NORMALIZED MTBF

TRANSMITTER TYPE	PEAK POWER (KW)	NORMALIZED FIELD MTBF (HRS)
ARSR-IA	4800.0	6549
AN/FPS-16	1000.0	ı I 47 0
AN/FPS-19	167.0	17430
AN/GPX-26	1.5	52104

The peak power and normalized field MTBF shows a straight line when plotted on log-log paper. This relationship, shown in Figure !, "Normalized MTBF's vs. Transmitter Peak Power", is described by the equation below.

It is of importance to note that the normalized MTBF is inversely proportional to the fourth root of the peak power while the maximum theoretical free-space detection range is directly proportional to the fourth root of the peak power; or, it may be concluded that if a designer should increase his peak power to obtain a desired detection range, the transmitter MTBF will probably decrease by the fourth root of the peak power.

A nomograph representative of the relationship discussed above was developed to facilitate the prediction of radar transmitter reliability. The normalized MTBF's were converted to normalized failure rate (e.g., failure rate = 1/MTBF) and used in this nomograph so that the resultant predicted reliability figure would be consistent with conventional methods of determining system reliability (failure rate) through the summarization of the



13

individual sub-system failure rates.

-1

The final technique for predicting the reliability of radar transmitters therefore requires using the nomograph shown in Figure 2 "Nomograph for Determining Radar Transmitter Failure Rate". This nomograph is based on the peak power and the number of active element groups that are anticipated to be used.

If the number of active element groups cannot be estimated during the very early design stages, it is suggested that an average value of 18 be used for the transmitter. The value of 18 was calculated as the average number of active element groups used in the four transmitters studied.

Relibility Prediction of the Receiver Function

Consideration of the pertinent characteristics associated with radar range detection led to the investigation of receiver sensitivity. Definite correlation was found between sensitivity (noise figure) and actual field MTBF's of the radar receiver included in this study. Other characteristics were investigated (e.g., bandwidth) however, no firm correlation was obtained.

Receiver sensitivity not only involves overall amplifier gain, but also the ability to extract or discriminate against self-generated noise as well as external noise. In operating radar receivers, this characteristic is usually measured directly by the minimum discernable signal (MDS) technique. However, in design and test, receiver noise is usually measured and expressed by one of two parameters; either the noise figure (NF) in db or the signal-to-noise ratio (S/N) also expressed in db.

The noise figure, which is presently more commonly used than the signalto-noise ratio, expresses the noise power level in a receiver front end as

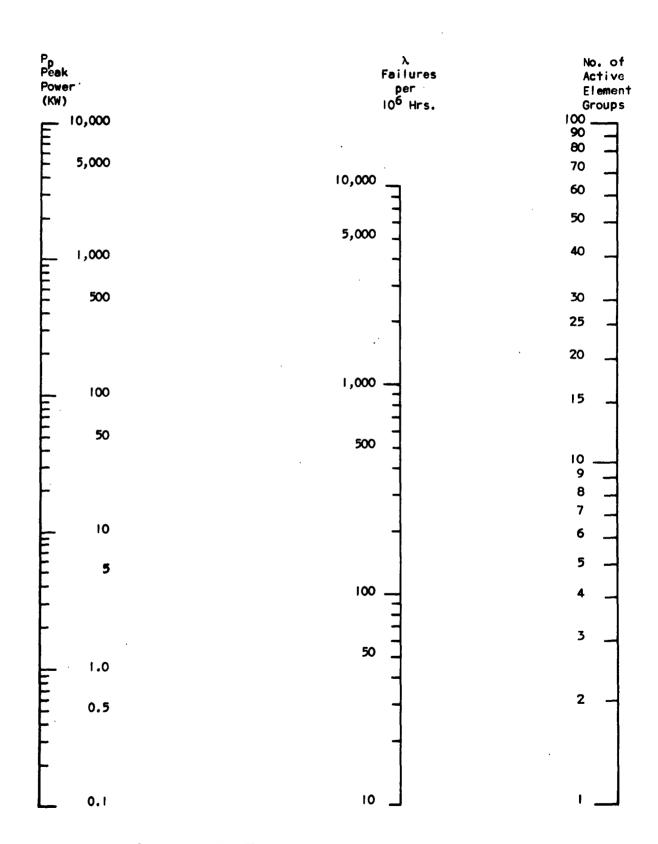


FIGURE 2. NOMOGRAPH FOR DETERMINING RADAR TRANSMITTER FAILURE RATE

a ratio of the internally generated noise (power) to the thermal noise power present in the same front end. Signal-to-noise ratio expresses the power ratio of a standard modulated output signal to the noise present in an unmodulated signal.

Since both noise figure and signal-to-noise ratio are important characteristics in a radar receiver, an analysis was conducted to determine their inter-relationship. An inverse relationship was indicated, or in other words, when one increases by some proportion the other decreases by the same proportion. Thus a 10% increase in noise figure would be equivalent in the signal-to-noise ratio scale of measurement (if this were physically possible) to a 10% decrease.

Because noise figure and signal-to-noise ratio are both a measure of radar receiver field performance, a correlation analysis concerning these two measurements and their respective normalized MTBF's was conducted.

Table 4, "Receiver Noise Figure/Signal-To-Noise Ratios and Normalized MTBF's" gives the actual field noise figure, the manufacturers maximum allowable noise figure, the manufacturers minimum acceptable signal-to-noise ratio, the actual field signal-to-noise ratio, and the normalized MTBF's for each receiver type. The maximum allowable NF and minimum acceptable S/N ratio are those values which when reacned would constitute failure in the applicable receivers. The actual field NF and S/N ratio are those which were measured in the field on operating equipments. All noise figures and signal to noise ratios are shown in db. The normalized field MTBF's were arrived at by the same procedure discussed previously for determining the transmitter normalized MTBF's.

TABLE 4

RECEIVER NOISE FIGURES/SIGNAL-TO-NOISE RATIOS AND NORMALIZED MTBF'S

RECEIVER TYPE	NORMALIZED MTBF'S HRS.	NOISE F	IGURE (db) MAXIMUM ALLOWABLE	SIGNAL TO	NOISE RATIO (db) MINIMUM ACCEPTABLE
AN/FPS-19	32241	8.3	10.0		
ARSR-IA	17485	8.0	8.5		
AN/FPS-16	19920	10.3	0.11		
AN/GPX-26	79176			14.6	11.0

In order to express all receiver noise comparisons to a common base it was necessary to use an index called the noise safety margin (NSM). This NSM is defined as either the ratio of the maximum allowable noise figure to actual noise figure minus one or the ratio of the actual signal-to-noise ratio to the minimum acceptable singal to noise ratio minus one, all in numerical units (not db). The NSM therefore can be expressed as follows:

NSM for receiver noise figure =
$$\frac{\text{Max. Allowable NF}}{\text{Actual NF}}$$
 -1 (3)

NSM for receiver signal to noise ratio =
$$\frac{\text{Actual S/N Ratio}}{\text{Min. Accept. S/N Ratio}}$$
 -1 (4)

The NSM can likewise be calculated from data expressed in db. Since noise measurements are in power, db equals 10 log 10 (Power Ratio). The ratio of the larger to the smaller noise measurement is then represented by the difference of the db values. The procedure then is to take the difference in db readings, divide by 10, take the antilog to the base 10 and subtract one.

When expressed in db units the above equations take on the following form:

NSM for receiver noise figure =

NSM for receiver signal-to-noise ratio =

The values of the Noise Safety Margin for all receivers are shown in Table 5
"Receiver Noise Safety Margins and Normalized Failure Rates". The normalized
failure rates were converted from the normalized MTBF's shown in Table 4.
The noise safety margins were calculated using the above equations (3 and
4) for expressing the NSM in numerical form (not db).

TABLE 5

RECEIVER NOISE SAFETY MARGINS AND NORMALIZED FAILURE RATES

RECEIVER TYPE	NORMALIZED FAILURE RATE PER 10 ⁶ HOURS	NOISE SAFETY MARGIN (NSM)
ARSR-IA	57.20	.122
AN/FPS-16	50.20	.175
AN/FPS-19	31.00	. 480
AN/GPX-26	12.60	1.290

The data shown in Table 5 was plotted on semilog paper with normalized failure rate on the linear scale and NSM on the log scale. Figure 3, "Receiver Noise Safety Margin Versus Failures Per 10⁶ Hours Per Active Element" illustrates the result of this plot. The resultant correlation equation for the straight line shown in Figure 3 is:

$$\lambda_{R} = 43.6 \log_{10} \frac{2.5}{NSM}$$
 (7)

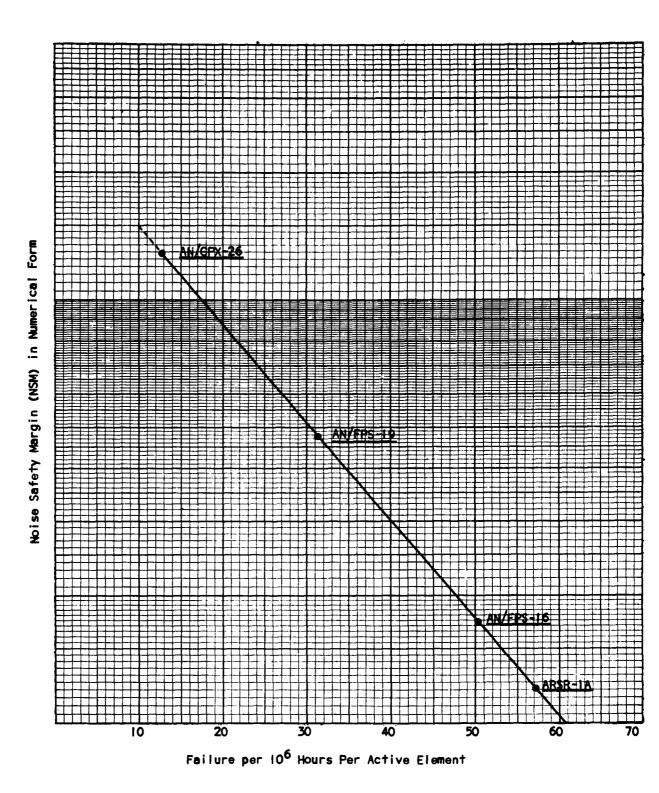


FIGURE 3. RECEIVER NOISE SAFETY MARGIN VERSUS FAILURES PER ACTIVE ELEMENT

where:

λ_R = Failure Rate per 10⁶ Hrs/Active Element Group (Normalized Failure Rate)

NSM = Noise Safety Margin (See equation 5 and 6)

The above equation utilizes the NSM which in turn is based on either the maximum allowable noise figure or the minimum acceptable signal-to-noise ratio and the actual noise measurements. It is felt that the designer would of necessity establish either the naximum allowable noise figure or the minimum acceptable signal to-noise ratios during the early design stages, however he would not have the actual noise measurements available to him.

If just the theoretical maximum design objective of a receiver were used by the designer in the correlation equation instead of the average noise measurements, a highly optimistic failure rate would be derived. This is because the correlation illustrated in Figure 3 was based on receivers which were in varied states of degradation with respect to noise; thus the NSM calculated from such data was considered to be lower than what would have resulted from measurements on new receivers immediately after installation. In order to make an estimate of the NSM of a radar receiver that is in the early design stage when using the procedure discussed above, the designer should halve the ideal initial db difference before anti-logs are taken and unity is subtracted (equations 5 and 6). This is based on the assumption that the correlation data represented a uniform linear spread between new or recently calibrated receiver quality (low noise) and a receiver with the worst case noise quality permissable before adjustment or part replacement would be necessary. The average quality would therefore be half way between the optimum and lower limit.

The resultant equation, therefore, for the prediction of the reliability of radar receivers is:

$$\lambda_{R} = 43.6 \log_{10} \frac{2.5}{NSM}$$
 (8)

where:

 λ_{R} = Receiver Failure Rate per 10^6 Hrs/Active Element Group

NSM = Noise Safety Margin

and:

NSM for Receiver Noise Figure (NF) =

NSM for Receiver Signal-To-Noise Ratio (S/N Ratio) =

The noise figure and signal-to-noise ratios shown in equations 9 and 10 are expressed in db.

The noise safety margin (NSM) may also be expressed in terms of minimum discernible signals (MDS) and minimum detectable signal (Smin).

Practical utilization of this equation for safety margins (NSM) above 1.5 is not recommended. From Figure 3 it can be observed that for a noise safety margin (NSM) of 2.5 a failure rate of zero is predicted, and for values greater, a negative failure rate results.

If desired, the curve shown in Figure 3 may also be used to predict radar receiver failure rate by calculating the receiver NSM, locating that value on the curve and reading the failure rate per 10⁶ hours per active element group and then multiplying this value by the estimated number of active element groups.

If the number of active element groups cannot be estimated during the very early design stages, it is suggested that value of 87 be used for the receiver. The value of 87 was calculated as the average number of active element groups used in the radar receivers that were studied.

Reliability Prediction of the Indicator Function

Indicator, as referred to herein, pertains to deflection-modulation (type A, etc.) and intensity-modulated (type PPI, etc.) cathode ray tube types. As previously discussed one of the most pertinent electrical characteristics of the indicator function is the visibility factor. Final analysis of this parameter showed that no apparent correlation between this factor and actual MTBF existed. Other characteristics were likewise investigated, however correlation could not be obtained. This does not mean that no correlation is possible, but that none was apparent from the data analyzed during this study.

However, an examination of expressions used for the visibility factor of indicators disclosed that the indicator which is almost entirely responsible for the purely geometrical aspects of the display problem, is sharing with the receiver the responsibility for the discernibility of the signals with respect to noise. Therefore an electrical parameter of the indicator is the ratio of received signal power to stored noise power. This parameter was covered in the receiver section of this study, because the receiver should include all items concerned with the received signal beginning at the antenna and ending with the input terminals of the indicator. The geometrical aspects of the indicator display, however, are concerned with the type of indicator; deflection-modulated display, A type, etc., or intensity-modulated display, PP1, etc.

The pertinent parameter considered in this study for the indicator function during the early planning stages of a radar equipment is the type of indicator desired. It is recommended therefore, that a constant failure rate be used along with the estimated number of active element groups in the prediction technique for the indicator function. A constant failure rate for a "PPI" or "A" type indicator is as follows:

"A" Indicator = 11.46 per 10^6 Hr./active element group

"PPI" Indicator = 9.96 per 10^6 Hr./active element group

These constant failure rates were calculated by Federal Electric Corporation from actual field data covering a one year period of operation.

If the number of active element groups cannot be estimated during the very early planning stages, it is suggested that an average value of 15 and 30 for the A and PPI indicators, respectively, be used. Values of 15 and 30 were calculated as the average number of active element groups used in the indicators shown in Table 1.

Reliability Prediction by Function for Radar Systems

The radar system reliability prediction technique is predicated upon its functions and their respective electrical parameters that should be known during the early planning stages of a radar equipment. Employing this philosophy, the MTBF of a radar is estimated through the summation of the functional failure rates as follows:

$$\lambda_{E} = (\lambda_{T} + \lambda_{R} + \lambda_{I}) \text{ per } 10^{6} \text{ hr.}$$
 (11) where:

 $\lambda_{\rm F}$ = equipment failure rate

 λ_{T} = transmitter failure rate

 λ_{R} = receiver failure rate

 λ_1 = indicator failure rate

The equipment estimated field Mean-Time-Between-Failure, MTBF, in hours is:

Estimated Equipment Field MTBF =
$$\frac{1}{\lambda_F}$$

In conclusion, to estimate the radar equipment field MTBF during its early planning stages, the following information for each function should be known.

1. Transmitter

- a. Peak power in kilowatts
- Estimated number of active element groups
 (if not known use 18)

2. Receiver*

- a. Maximum allowable receiver noise figure
- b. Anticipated actual receiver noise figure
- c. Estimated number of active element groups(if not known use 87)

3. Indicator

Number of estimated active element groups
 (if not known use 15 or 30 depending on the type indicator used)

*Instead of receiver noise figure, the signal-to-noise ratio (S/N), or the minimum detectable signal power (S_{min}) and minimum discernible signal power (MDS) may be used.

Verification of Reliability Prediction by Function Technique for Radar

The technique for predicting the reliability of radar by function was

verified by estimating the MTBF of a selected pulsed radar using the subject
technique and comparing resultant MTBF's to those calculated from operational

field data. This radar has been designated as "Radar A" since it is currently a classified radar. The prediction and comparison of the reliability of this radar is shown in detail below.

Radar "A" Verification

Radar "A" is maintained and operated by Federal Electric Corporation and the operational and failure data concerning this equipment was compiled from one year (I September 1961 to 31 August 1962) of field operational data.

In order to estimate the MTBF of Radar "A", the required information for each function was tabulated as follows:

!. <u>Transmitter</u>

- a. peak power in kilowatts = 500KW
- b. number of active element groups = 21

2. Receiver

- a. maximum affowable minimum discernible signal (MDS) = -95 dbm (as specified by Mfg.)
- v. minimum detectable signal $(S_{min}) = -98 \text{ dbm}$ (as measured during field operation.)
- c. number of active element groups = 48

3. Indicator

- a. A Type with 36 active element groups
- b. PP! Type with 44 active element groups

The manufacturer of this radar does not specify a receiver noise figure or signal-to-noise ratio. In this case the noise safety margin (NSM) is the ratio of the minimum discernible signal (MDS) power to the minimum detectable singa: (S_{min}) power minus one in numerical units. These figures

of multi should be known during the early planning stages of a radar equipment.

The receiver NSM is therefore calculated (expressed in dbm units) as follows:

NSM = anti-log
$$\frac{S_{min} - MDS}{10}$$
 -1
= (anti-log .3) -1
= 1.9956-1
= .9956

It should be noted that the quantity ten (IO) is used as the divisor, rather than twenty (20), because of the fact that actual measured values for S_{\min} were available.

Having determined the necessary functional informational requirements of Radar "A", the estimated equipment MTBF is developed as follows:

Step I - Transmitter estimated failure rate, λ_T

The transmitter failure rate (λ_T) is estimated from the nomogram shown in Figure 2 for a peak power of 500KW and 21 active element groups. Thus, the transmitter failure rate is λ_T = 1840 per 10⁶ hours.

Step 2 - Receiver estimated failure rate λ_R

The receiver failure rate (λ_R) is estimated from the following equation: $\lambda_R = 43.6 \log_{10} (2.5/\text{NSM}) \text{ per } 10^6 \text{ hrs/active element group}$ where:

 λ_R = receiver failure rate per 10^6 hrs.

NSM = noise safety margin.

Using the calculated noise safety margin of 0.9956 and 48 active element groups, the receiver failure rate (λ_R) is:

 $\lambda_{\rm p}$ = 836.8 per 10^6 hours.

Step 3 - Indicator estimated failure rate λ_{\parallel}

Radar "A" employs both A and PPI type indicators. The total indicator failure rate, λ_{\parallel} , is therefore, the sum of the individual indicator failure rates, calculated as follows:

$$\lambda_1 = \lambda_A + \lambda_P$$

Where:

**

 λ_1 = indicator failure rate

 λ_A = 11.46 per 10⁶ hrs. x 36 (For "A" type indicator)

 $\lambda_{\rm P}$ = 9.96 per 10⁶ hrs. x 44 (For PPI type indicator)

thus:

 $\lambda_{\Delta} = 412.56 \text{ per } 10^6 \text{ hrs.}$

 $\lambda_{\rm P} = 438.24 \ {\rm per} \ 10^6 \ {\rm hrs.}$

 $\lambda_1 = 412.56 + 438.24 = 850.8 \text{ per } 10^6 \text{ hrs.}$

Step 4 - Estimated Failure Rate $\lambda_{\textrm{F}}$ and MTBF

The failure rate, λ and estimated field MTBF for Radar "A" is determined from the sum of the individual failure rates previously calculated in Steps 1, 2 and 3 immediately above.

 $\lambda_F = \lambda_T + \lambda_R + \lambda_I \text{ per } 10^6 \text{ hours.}$

where:

 λ_F = system failure rate

 λ_T = transmitter failure rate

 λ_{D} = receiver failure rate

 λ_i = indicator failure rate

Inserting the failure rates previously calculated for the transmitter, receiver and indicator of 1840.0 per 10^6 hrs. 836.8 per 10^6 hrs., and 850.8 per 10^6 hrs., respectively, yields an equipment failure rate of 3527.6 per 10^6 hrs. The estimated field MTBF of Radar "A" is:

MTBF =
$$I/\lambda_F$$
 = 283 hrs.

A comparison of the estimated field MTBF of 283 hours and the estimated MTBF's of the functions of Radar "A" with the actual recorded MTBF's covering one year of operational data, is shown in Table 6, "Comparison of Radar "A" Predicted and Actual MTBF's".

TABLE 6

COMPARISON OF RADAR "A" PREDICTED AND ACTUAL MTBF'S

RADAR A FUNCTION	PREDICTED MTBF (HRS)	ACTUAL MTBF (HRS)
Transmitter	543	570
Receiver	1195	1140
Indicator A	2424	4559
Indicator PPI	2282	1150
Equipment	283	268

The field data for Radar "A", while covering one year of calendar time, showed actual operating time for all equipments of approximately 4559 hours. During this time 17 failures were reported resulting in an MTBF estimate of:

MTBF =
$$\frac{4559}{17}$$
 = 268 hours

The 90% confidence band² for this MTBF value was calculated as follows:

Lower Limit = L =
$$\frac{2T}{\chi^2}$$
 $\propto/2$; $2r + 2$ (12)

Upper Limit = U =
$$\frac{2T}{\chi^2}$$
 (13)
(1- α /2); 2r

²Method of calculating the 90% confidence interval was obtained from "Estimation from Life Test Data", B. Epstein, IRE Transactions on Reliability and Quality Control, Volume RQC-9, (April 1960).

where:

2r

Т = total hours of operation = 4559 hours

 \propto = percentage point at which the χ^2 value is needed and in this case equals unity minus the confidence desired (90%) expressed as a decimal or l - .90 = .10

= value from Chi-square distribution table for desired values of and appropriate cegrees of freedom

= number of failures observed = 17

2r+2 = degrees of freedom for χ^2 in equation 12 above = 2(17) + 2 = 36 = degrees of freedom for χ^2 in equation 13 above = 2(17) = 34

From the Chi-Square tables:

 χ^2 for equation 12 = 51.000 and χ^2 for equation 13 = 21.662

Thus the lower limit (L) of the band = $\frac{4559}{51.000}$ = 179 hours and the upper limit (U) of the band = $\frac{4559}{21.662}$ = 420 hours

To summarize, the operational field data shows an MTBF for Radar "A" of 268 hours with a 90% confidence band of 179 to 420 hours around it. The estimated MTBF of 283 hours falls well within this 90% confidence band. If the number of active element groups per function type could not have been estimated and the suggested values of 18 for the transmitter, 87 for the receiver, 15 for an A type indicator and 30 for a PPI indicator were used, the estimated MTBF would have been 274 hours which agrees quite favorably with the predicted value of 283 hours and the value of 268 hours derived from the field operational data.

Ground Communication Systems

The development of a reliability prediction technique for ground communication systems followed the basic approach utilized for radar, that is the correlation of field operational and failure data (MTBF's) with pertinent electrical parameters. The technique is applicable to both tropospheric (tropo) scatter communication and line-of-sight microwave systems. However, tropo systems have been used in the following discussions concerning the development of the prediction technique.

Basic Design Parameters of Ground Communication System

Noise and power measurements in a radar system differ materially from those in a communication system in that the former is essentially a single pulse retrieval system in which noise margins are easily determined. Communication systems on the other hand, utilize analog amplifiers as opposed to pulsed in radar, in which the information is more closely interspersed with transmitter and receiver noise. However the margins measured, (e.g., S/N, MDS) remain the same. This holds even when using digitalized information because the retrieval is continuous and processes trains of un-associated, non-synchronous pulses at unrelated intervals.

In early design of a communication system there are a number of factors such as site location, distance between sites, frequency spectrum allocation of existing communication mediums, etc., which must be considered. These factors have a direct effect on parameters such as frequency, transmitting power, receiver bandwidth, etc. Usually the basic starting point in a communication system design is the carrier-to-noise ratio, (C/N).

A typical formula 3 in decibels that can be used by system designers to determine C/N during the early planning stages of a communication system consisting of one hop is as follows:

$$C/N = P_{+} - L_{+} + G_{+} - L_{fs} - L_{BH} + G_{R} - L_{R} + G_{FM} + D_{C} - N + B + K_{N}$$
 (14)

where:

P = transmitter operational power

L_r = transmission - line losses

G₊ = transmitting antenna gain over an isotropic path

L_{fs} = free space loss

 L_{RH} = beyond the horizon loss

 G_R = receiving antenna gain (same as G_+)

L_D = receiver line losses (same as L₊)

G_{FM} = FM gain for deviation ratio (SSB=1.0)

D = receiver diversity combination gain

N = receiver noise figure in db below I watt

B = $10 \log b_{kc} + 10$ where b is the receiver bandwidth in kilocycles

 $K_N = 0.01 \text{ KT where } K = \text{Boltzmann's constant and } T \text{ is } 293^{\circ}$

Examination of each item in the above equation illustrates the role that design parameters play in a system design. In general, the specific parameters used during the early planning stages are to a great extent dependent

K. A. Norton - Transmission Loss in Radio Propagation - Proceeding's of IRE, Volume 41, Fages 146 - 152, January 1953.

[&]quot;Reference Data for Radio Engineers", Fourth Edition, International Telephone and Talegraph corporation, July 1957, Page 756.

K. A. Norton and A. C. Omberg - Maximum Range of a Radar Set - Proceeding's of IRE, Volume 35, Pages 4-24, January 1947.

upon the desired C/N ratio of the communication system. As such the functional requirements of transmitter, exciter, receiver and multiplex, using the pertinent system design parameters shown above were analyzed for possible correlations with their actual field MTBF's.

Approach to Reliability Prediction of Ground Communication Systems

Several tropospheric scatter communication systems were selected for analysis. The actual field MTBF's of the transmitter, exciter, receiver and multiplex functions were calculated from field operational data available within Federal Electric Corporation. These systems comprised a total field operating population of 224 transmitters, 222 exciters, 414 receivers and 100 multiplexers. A list of the systems analyzed and the periods of time covered by the field data is shown in Table 7, "Tropospheric Communication Systems Analyzed".

TABLE 7
TROPOSPHERIC SCATTER COMMUNICATIONS SYSTEMS ANALYZED

SYSTEM NOMENCLATURE	FIELD DATA TIME PERIOD
AN/FRC-45(V) IKW	l January 1961 to 31 December 1961
AN/FRC-45(V) 6.5KW	I January 1961 to 31 December 1961
AN/FRC-39A(V) IKW	l August 1961 to 30 April 1962
AN/FRC-39A(V) 7KW	l August 1961 to 30 April 1962
AN/FRC-39A(V) 50KW	August 1961 to 30 April 1962
REL IKW	l January 1959 to 30 November 1959
REL 50KW	I January 1959 to 30 November 1959

The actual field MTBF's calculated for the transmitter, exciter, receiver and multiplex functions for each of the three systems are given in Table 8, "Actual MTBF's of Tropo Functions".

TABLE 8

ACTUAL MTBF'S OF TROPO FUNCTIONS

COMMUNICATION SYSTEM	FUNCTION	ACTUAL MTBF (HRS)
AN/FRC-45(V)	Transmitter (6.5KW) Transmitter (IKW) Exciter Receiver (Conventional) Multiplex	3504 5553 1061 1033 756 (24 Voice Channels)
AN/FRC-39A(V)	Transmitter (7KW) Transmitter (1KW) Transmitter (50KW) Exciter Receiver (Parametric) Receiver (Conventional) Multiplex	2366 3082 2333 2184 2010 4417 No Data Available
R. E. L.	Transmitter (5.5KW) Transmitter (!KW) Exciter Receiver (Convertional) Multiplex	1483 5428 817 673 756 (24 Voice Channels)

A number of individual correlation studies were made singularly on the separate functional levels of transmitter, exciter, receiver and multiplex, however, no significant results were obtained.

Since the communication systems studied were composed of various combinations of these functional units (e.g., receivers) it was necessary to examine the relationship of these units when operating together as an overall system. When this was accomplished analysis showed correlation between the combined MTBF of transmitter, exciter, receiver and the C/N ratio.

No firm correlation was detected in the data compiled on the multiplex function; therefore it was treated separately.

Reliability Prediction of the Transmitter, Exciter and Receiver Function Although the characteristic of C/N ratio was found to exhibit a correlation with the overall MTBF's of the functional levels of combined transmitter, exciter and receiver, it was necessary to analyze these combinations as a system, particularly because the systems studied contained redundant functional configurations and utilized diversity techniques. Analysis thus determined whether or not the failure of a particular function would result in a system failure. For example in the AN/FRC-45(V), IKW, operating on a one hop condition, with dual diversity reception, one transmitter, one exciter and two receivers are employed to provide communication. A simplified block diagram of this configuration is shown in Figure 4. This figure does not include any indication of redundant functions normally associated with the AN/FRC-45(V). The failure of any one of these functions, including either one of the receivers, would reduce the C/N ratio to a level where system failure would be encountered. The communication systems failure rate (λ_{C}) for this configuration was therefore calculated as follows:

$$\lambda_{\rm C} = \lambda_{\rm T} + \lambda_{\rm E} + 2\lambda_{\rm R}$$
 (15) where:

 λ_{T} = Failure Rate of One Transmitter

 $\lambda_{\rm F}$ = Failure Rate of One Exciter

 λ_{R} = Failure Rate of One Receiver

Another example is a one-hop scatter communication system operating in quadruple-diversity reception, e.g., the AN/FRC-39A(V), where two transmitters and two exciters and four receivers are used in normal operation.

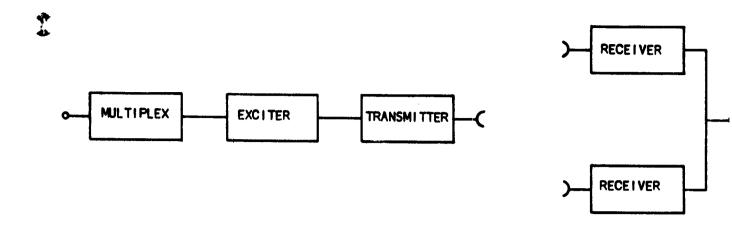


Figure 4 AN/FRC-45(V) | KW, Simplified Diagram

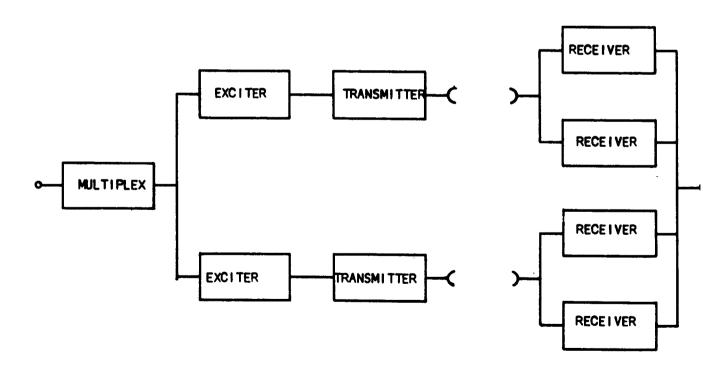


Figure 5 AN/FRC-39A(V) I KW, Simplified Block Diagram

A simplified block diagram of this configuration is shown in Figure 5. The redundant functions normally used with the AN/FRC-39A(V) are not included in this figure. The loss of one receiver in this type of communication will usually not constitute a system failure and the simultaneous loss of more than one receiver is remote. Under these conditions the receiver failure rate λ_R has relatively little effect on the overall system reliability and effectively drops out. The failure rate for this type of communication system configuration (λ_C) was therefore calculated as follows:

$$\lambda_{C} = 2\lambda_{T} + 2\lambda_{E}$$
 (16) where:

 λ_{T} = Failure Rate of One Transmitter

 $\lambda_{\rm F}$ = Failure Rate of One Exciter

Similar system failure rates were developed for the other configurations studied and all were converted to MTBF figures. Table 9 "Ground Communication Systems Calculated C/N Ratios and MTBF's", shows these MIBF figures as well as the calculated C/N ratio for each of the systems.

These MTBF values were compared to their respective C/N ratios by means of rank correlation methods in order to determine whether further correlative efforts were warranted. The method used was Spearman's rank correlation coefficient. Table 10, "System C/N Ratio and MTBF Rank Order Correlation" presents the results of this correlation.

⁴S. Siegel - Nonparametric Statistics for the Behavioral Sciences, McGraw-Hill, 1956, Page 202.

TABLE 9

GROUND COMMUNICATION SYSTEMS CALCULATED C/N RATIOS AND MIBF'S

COMMUNICATION EQUIPMENT	CALCULATED MTBF (HRS)	CALCULATED C/N RATIO (db)
AN/FRC 45(V) IKW	326	62.1
AN/FRC-45(V) 6.5KW	316	45.5
AN/FRC-39A(V) IKW	639	68.1
AN/FRC-39A(V) 7KW	568	58.4
AN/FRC-39A(V) 50KW	382	54.8
REL IKW	184	45.1
REL 5.5KW	210	54.3

TABLE 10

SYSTEM C/N RATIO AND MTBF RANK ORDER CORRELATION

EQUIPMENT TYPE	C/N RATIO	RANK	MTBF	<u>RANK</u>	(A RANK)2
REL IKW	45.1	1	184	1	0
AN/FRC-45(V) 6.5KW	45.5	2	316	3	ı
REL 5.5KW	54.3	3	210	2	1
AN/FRC-39A(V) 50KW	54.8	4	382	5	ļ
AN/FRC-39A(V) 7KW	58.4	5	568	6	ì
AN/FRC-45(V) IKW	62.1	6	326	4	4
AN/FRC-39A(V) IKW	68.1	7	639	7	0
			Total	(△ Rank)²	8
				j	

As can be seen from Table 10, the total of $(\triangle Rank)^2$ is equal to a value of 8. The equation for the Spearman rank order correlation coefficient is:

$$h_{S} = 1 - \frac{6 \sum_{N(N^2-1)}^{2}}{N(N^2-1)}$$
 (17)

where:

 A_s = Spearman rank correlation coefficient

d² = the square of the difference in rank of a paired observation

N = number of paired observations

Therefore Λ_s is found by inserting the required values from Table 10 ($\xi d^2 = 8$ and N = 7) into equation 17 as follows:

$$A_{s} = 1 - \frac{6 \times 8}{7(7^{2}-1)}$$

$$= 1 - \frac{48}{336}$$

$$= 1 - .143$$

$$= 0.857$$

A value of h_s , equal to 0.857 results in a confidence level of 98.85%. This means that there is a 98.85% probability that a correlation exists between the C/N Ratio and MTBF data shown in Table 9. The 98.85% confidence level is derived at by determining the number of permutations of the numbers 1 through 7 which result in a sum of squares of rank difference, $(\triangle \text{Rank})^2$ of 8 or less. This number was determined to be 58. Since all the possible permutations of 7 are equal to 7 factorial, a total of 5040 or 7! permutation can exist. Of these 5040 only 58 can result in a sum of $(\triangle \text{Rank})^2$ equal to 8 or less. Therefore the confidence level is equal to:

$$=\frac{1-58}{5040} \times 100 = 98.85\%$$

Since \mathcal{K}_S is a positive number, the slope will be positive (upwards to the right). The C/N ratio and corresponding MTBF's for each system were plotted on linear paper. A straight line plot appeared to be the most suitable fit for the range of data covered and a least squares line was determined. The resultant plot is shown in Figure 6, "Relation—ship Between C/N Ratio and MTBF for Combined Transmitter, Exciter and Receiver Functions of Ground Communication Systems". The equation for the least squares line is:

$$MTBF_{TFR} = 14.9 (C/N Ratio - 30.3)$$
 (18)

It should be noted that the predicted MTBF should not be estimated for these functions for C/N ratios less than 40 db.

The final technique for estimating the predicted MTBF of the transmitter, exciter and receiver for Ground Communication Systems utilizes either the plot shown in Figure 6 or the above equation. As mentioned previously this technique is applicable to both Tropospheric Scatter and Line-of-Sight Microwave Communication Systems.

This technique presumes that the designer had imposed a number of limiting conditions, the combination of which must be arranged or tailored for best performance and optimum system reliability. For instance, he generally knows within reasonable limits the transmitter and receiver feeder line losses. With fixed site positioning, the free space and beyond-the-horizon losses are readily obtained from design tables. Receiver diversity improvement is common knowledge and experience has dictated frequency

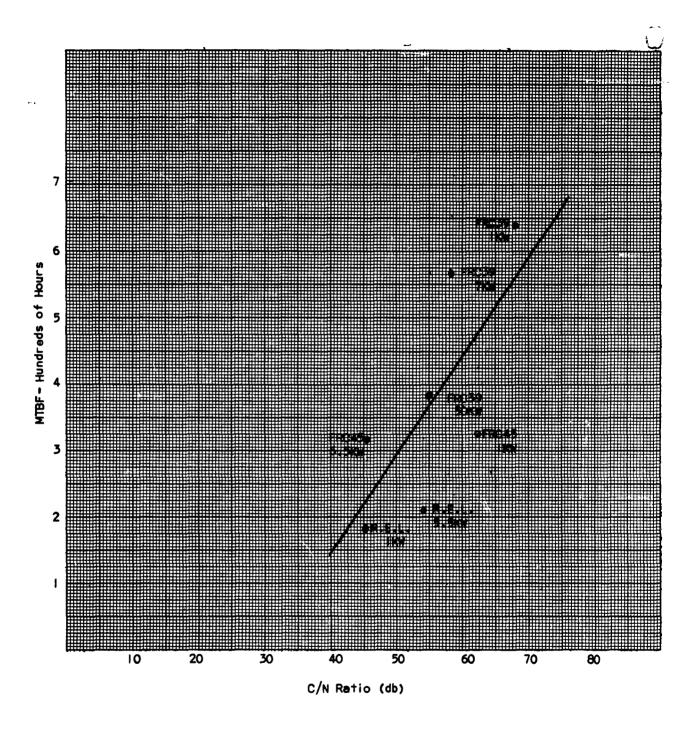


FIGURE 6. RELATIONSHIP BETWEEN C/N RATIO AND MTBF FOR COMBINED TRANSMITTER, EXCITER AND RECEIVER FUNCTIONS OF GROUND COMMUNICATION SYSTEMS.

modulation characteristics plus receiver and inherent noise characteristics. He then chooses sufficient transmitter power to attain a reasonable C/N ratio. This defines and brackets the operating point upon Figure 6 so that MTBF's can be directly determined.

The MTBF value obtained from the above technique can be converted to failure rate of the Transmitter, Exciter and Receiver functional combination (TER) by means of the following conventional formula:

$$\lambda_{\text{TER}} = \frac{1}{\text{MTBF}_{\text{TER}}}$$
 (19)

Reliability Prediction of the Multiplex Function

There were only two basic types of multiplex equipment included in the communication systems studied. The 45BXT2 multiplex was employed in all configurations of the AN/FRC-45(V) and REL, while the AN/FRC-39(V) configuration used an L&K Carrier type of multiplex. Actual field MTBF's however could only be determined on the 45BXT2. Paucity of operational and failure data obviated analytical studies for the L&K Carrier type multiplex.

It was not possible to develop correlation between any pertinent electrical characteristics of the multiplex function and actual MTBF's. Various correlation attempts were made, however lack of a sufficient amount of data from various types of multiplex equipments precluded the development of any valid technique.

Since there was a relatively large amount of data available on the 45BXT2 and since no correlation could be detected, it was decided that the treatment of multiplex would be directed along the same lines as that of the radar indicator function. In other words utilize actual field failure

and operational data so as to arrive at an MTBF that would be representative of the multiplex function.

Since multiplex equipment usually employs twelve (12) channel groups, failure rates were developed for this combination based on data collected on the 45BXT2. The failure rate of a twelve (12) channel group (λ_{12CG}) was calculated to be 661 failures per 10^6 hours. The 45BXT2 uses two channel groups, therefore, the failure rate of the 45BXT2 is twice the figure shown above or 1322 failures per 10^6 hours. The MTBF for the 45BXT2 is 756 hours.

Although the above failure rate and MTBF data developed for the multiplex function is based on data from one particular type of multiplex, the 45BXT2, it may be used as a ballpark estimate for other types of multiplex.

In order to determine the relative worth in applying these figures they were compared to MTBF's on two completely different types of multiplex equipments, the AN/TCC-7, which is a vacuum tube type multiplex and the MC-50 which is a transistorized multiplex. It should be noted that both of these equipments have different modulation plans.

The failure rates for these two equipments were calculated using the stress method for parts outlined in RADC Reliability Notebook TR-58-III, 31 December 1961, Section 8. Using the modulation plan for the equipments and setting the number of twelve (12) channel groups equal to two, and using the predicted failure rates from TR-58-III, the MTBF for the AN/TCC-7 was determined to be 880 hours and that of the MC-50 to be 659 hours. When the value of 756 hours is compared to these results it was found that it is approximately 16% above that predicted for the AN/TCC 7 and approx-

imately 15% below that predicted for the MC-50.

It was therefore concluded that using the above failure rate for a twelve (I2) channel groups and the number of twelve (I2) channel groups, a reasonably accurate ballpark estimate can be developed for the multiplex functions even though they may have different modulation plans.

The failure rate of multiplex (λ_M) is therefore estimated by employing the following equation:

$$\lambda_{M} = (\lambda_{i2CG}) N$$

where:

 λ_{12CG} = the failure rate per 10^6 hours for one twelve (12) Channel Group (CG) and is equal to 661 failures per 10^6 hours.

N = the number of operating twelve (12) channel groups.

Reliability Prediction by Function for Ground Communication Systems

The prediction of ground communication system reliability is based on the reliability of its component functions. The MTBF of a communication system therefore is estimated through the summation of its functional failure rates as follows:

$$\lambda_{C} = \lambda_{TER} + \lambda_{M}$$

where:

 $\lambda_{\widehat{C}}$ is the farrure rate of the system

 $\lambda_{\mbox{TER}}$ is the failure rate of the transmitter, exciter and receiver

 $\lambda_{\boldsymbol{M}}$ is the failure rate of the multiplex

The estimated ground communication system MTBF = $1/\lambda_{r}$

To estimate the MTBF of a ground communication system the following information for the functions should be known.

- 1) Transmitter, Exciter and Receiver Function:
 - a) calculated or pesired C/N ratio
- 2) Multiplex Function
 - a) the desired number of operating
 twelve (12) channel groups
 - b) the failure rate of one twelve (12) channel group = λ = 661 failure per 10⁶ hours

Verification of Reliability Prediction by Function Technique for Ground Communication Systems

The technique for predicting the reliability of ground communication systems by function was verified by estimating the MTBF of a tropospheric scatter communication system, the AN/FRC-47(V), and comparing this value to actual MTBF's that were calculated from operational and failure data. Since the technique is applicable to Line-ot-Sight Microwave systems the technique was also applied to one such system, the AN/TRC-29 and the resultant MTBF's compared to those calculated from actual operational and failure data.

AN/FRC-47(V) Verification

The AN/FRC-47(V) is a UHF Single Sidehand scatter communication equipment. Field data used to calculate the MTBF's of this equipment covered approximately a one year period of operation.

In order to estimate the MTBF of the AN/FRC-47(V) the following information for its functions was obtained:

- 1. Exciter, Transmitter, and Receiver Function
 - a) calculated or desired C/N ratio = 41.8db
- 2. Multiplex Function
 - a) the number of twelve channel groups (N) = 2
 - b) the failure rate of one twelve channel $group = \lambda_{12CG} = 661 \text{ failures per } 10^6 \text{ hours}$

Having determined the necessary information for the functional requirements of the AN/FRC-47(V) it's estimated equipment field MTBF was calculated by the following steps.

Step I - Transmitter, Exciter and Receiver Function Failure Rate

The transmitter, exciter and receiver combined field MTBF is first calculated from the following equation:

MTBF =
$$14.9 (C/N - 30.3)$$

= $14.9 (41.8 - 30.3)$
= 171 hours

The MTBF of 171 hours may be expressed in failure rate (λ_{TER}) per 10^6 hours, as follows:

$$\lambda_{TFR} = 10^6 / MTBF = 5848 / 10^6 \text{ hours}$$

Step 2 - Multiplex Function Failure Rate

The failure rate of the multiplex function is calculated from the following equation:

$$\lambda_{M} = (\lambda_{12CG}) N$$

$$= 661 \times 2$$

$$= 1322 \text{ failures per } 10^{6} \text{ hours}$$

Step 3 - AN/FRC-47(V) Equipment MTBF

The MTBF of the AN/FRC-47(V) is determined by taking the reciprocal of the sum of the failure rates previously calculated for its functional requirements, as follows:

$$\lambda_{\rm C} = (\lambda_{\rm TER} + \lambda_{\rm M}) \text{ per } 10^6 \text{ hours}$$

$$= (5848 + 1322)$$

$$= 7170/10^6 \text{ hours}$$

The AN/FRC-47(V) estimated field MTBF, therefore is: $MTBF = \frac{1}{\lambda_C} = 139 \text{ hours}$

A comparison of the estimated field MTBF of 139 hours and the estimated MTBF's of the functions of the AN/FRC-47(V) equipment, with actual MTBF's covering a one year period of operation is shown as Table II, "Comparison of AN/FRC-47(V) Predicted and Actual MTBF's". The actual MTBF of 219 hours was based on an operational time of 8760 hours.

TABLE II

COMPARISON OF AN/FRC-47(V) PREDICTED AND ACTUAL MTBF'S

AN/FRC-47(V) FUNCTION	PREDICTED MTBF (HRS)	ACTUAL MTBF (HRS)
Transmitter, Exciter and Receiver	171	219
Multiplex	756	756
Equipment	: 39	170

A "confidence" band was assigned only to the transmitter-exciterreceiver MTBF since the multiplex used in this system is the same as
the one from which the prediction procedure for multiplexes was derived.
Using the standard chi-squared method, previously discussed on page 28
the 90% confidence band for the TER combination is 168 to 290 hours.
Combining this with an absolute value of 756 hours for the multiplex
results in a system MTBF 90% confidence band of 137.5 to 209.6 hours.
The predicted equipment MTBF of 139 hours just falls within this band.

Comparison of Predicted and Actual MTBF's for AN/TRC-29 Microwave System
The actual MTBF's for the AN/TRC-29 Microwave were obtained from "Summary Report on the Reliability Study of Radio Set AN/TRC-29 and Associated Equipments" prepared by Federal Electric Corporation. This study was based on the observation of actual field performance of the AN/TRC-29 and associated equipments during a six month period of operation. The actual MTBF data extracted from the above report were compared with that calculated using the prediction techniques discussed above. The results of this comparison are shown in Table 12 "Comparison of AN/TRC-29
Predicted and Actual MTBF's". It should be noted that the nomenclature for the multiplex equipment used in the above study is the AN/TCC-13.

TABLE 12

COMPARISON OF AN/TRC-29 PRED:CTED AND ACTUAL MTBF'S

AN/TRC-29 FUNCTION	•	PREDICTED MTBF	ACTUAL MTBF
TER		368	480
MUX 7.56		756	577
Equipment ^		248	261

The multiplex MTBF was based on 21 failures in 12,117 hours of operation, while the rest of the system was based on 121 failures in 58,101 hours of operation. Since the 21 failures determine the limit of confidence, a conservative estimate of confidence on system MTBF was made by converting the 121 failures to the equivalent number which would occur in 12,117 hours. This came out to be 25. The total number of equivalent system failures therefore is 46 (21 + 25) in 12, 117 hours and the system 90% confidence band is 206 to 343 hours, based on the previously discussed chi-square technique of estimating confidence bands on MTBF's. The predicted 248 hours falls well within this band.

Data Processing Systems

The initial approach taken for the development of a technique for predicting the reliability of electronic data processing (EDP) systems was similar to that taken for radar and ground communication systems. That is, several representative EDP systems were investigated in order to determine logical functional breakdowns and associated characteristics which appeared suitable for correlation to actual MTBF's. It was concluded that no single functional characteristic could be obviously selected for computers and associated equipments making up an EDP complex which would be applicable for correlation to MTBF. The approach taken was to isolate functional areas and to seek out those particular characteristics which logically affect reliability.

The first step taken was to divide an electronic data processing system into two general operational areas: 1) central processor and, 2) peripheral equipment. The central processor is composed of the functional units which control the operation of the input, output and buffer equip-

ment, mathematically operates on the data it receives, extracts required data from memory, stores the resultant data, and ultimately controls the transmission to the output equipment, while peripheral equipment was defined as the input, output and other equipments not under the direct control of the central processor.

The principle functions of a central processor therefore are: 1) control,

2) transmit, 3) synchronize, 4) storage and 5) logic, while the principle
functions of the peripheral equipment are input, output and storage.

There usually are various means of performing a function. The fact that
there are numerous ways of performing a function in EDP systems (for
example, input can be accomplished by card, paper tape, magnetic tape,
etc.), whould be considered in the development of an overall procedure for
predicting the reliability of EDP systems.

The next step was the determination of functional characteristics which were suspected of demonstrating correlation with MTBF. The speed of information delivery, which is usually measured in thousands of bits per second, appeared to be a pertinent characteristic of both the input and output function. The read time and write time, usually expressed as thousands of character records per millisecond, were other characteristics selected for the input and output functions respectfully. The access or start/stop time in milliseconds was considered pertinent to the storage function. Characteristics of the central processor were the computational time of the logic function in microseconds and the cycle-operation time for the control function, also in microseconds.

The planned approach was to select a representative number of different physical types of functional units and to attempt correlation of the

functional characteristics such as discussed above with actual MTBF's. It was suspected that in the case of some functions (e.g., input) the means (e.g., magnetic tape vs. paper tape readers) would have an influence on resultant MTBF. Therefore analysis was to be directed not only toward a function but also toward the means for performing the function.

Another consideration which was believed to have a possible effect on the correlation was the number of characteristics or features inherent in a particular function. For example, features such as checking capability might influence resultant MTBF's. In the same manner, features which might be incorporated in central processor functions such as double precision computation, floating point arithmetic, multiple addressing, word length, etc. were also felt worthy of consideration.

The general approach described above could not be implemented since field and operational data required for correlation could not be obtained in the quantity and orientation needed for correlation. As a result, no firm technique could be established at this time for predicting the reliability of electronic data processing systems by function.

SECTION 3

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This study has resulted in firm techniques for predicting the reliability of radar and ground communication (tropospheric scatter and line-of-sight microwave) systems as well as investigations related to the development of similar techniques for electronic data processing systems. It was recognized during the earlier part of this contract that the scope of the program would not permit the investigation of all possible functional levels of systems, treated individually or in combination. Therefore, the general approach followed was to develop techniques for complete systems by concentrating on the principal sub-systems, at the highest level possible.

In brief, the techniques for radar and ground communication systems are based on estimating the reliabilities of the principal sub-systems or functional levels using pertinent electrical characteristics of the functional levels. The reliability of the system is then arrived at by combining the reliability of these functional levels.

Investigations were also directed toward the development of techniques for predicting the reliability of electronic data processing (EDP) system during the early design stages. The approach taken was the establishment of appropriate functional levels and the isolation of pertinent characteristics which could be correlated with actual MTBF figures. Difficulty was encountered in obtaining appropriate field operational and failure data on EDP systems required for correlation. As a result no firm technique could be developed for predicting reliability of EDP systems.

In summary it is believed that the technique for predicting the reliability of radar and ground communication systems in the early design stages is a significant contribution to the state-of-the-art. The principal advantage of these techniques is that they provide a method of estimating the reliability of a system early in the design cycle when there is a relatively small amount of detailed information available.

It should be pointed out that these techniques are not limited to systems in the early design stage. They can be applied to systems at any point above this stage up to and including production models. However, it is believed that the results of predictions based on these techniques for systems that have progressed from the early design stage would not be as accurate as those derived from the more rigorous, but time consuming methods outlined in the RADC Reliability Notebook TR-58-!!!

The proposed approach for correlation of electronic data processing system data shows promise and should be continued and finalized, however, it will be necessary to obtain actual operational and failure data in sufficient quantity and on a representative number of different types of systems to accomplish this goal. Once this is accomplished, it will be possible to estimate the reliability of a command or control complex, based on the individual reliabilities of its principal systems, during the early planning stages.

It should be noted that supplementary prediction techniques for auxiliary equipments included in a command complex, as well as for functional levels of radar and ground communication systems not covered in this study, would further refine and improve the overall prediction of reliability of a command or control complex.

Once valid techniques have been developed for the prediction of system reliability, the next logical step is to develop similar techniques for predicting system maintainability in the early planning stages so that system availability, which is in fact a more valuable figure-of-merit to the user than reliability or maintainability, can be estimated.

Recommendations

Based upon the results of investigations and analyses performed in this program, the following recommendations are made.

- I. The techniques for predicting the reliability of radar and ground communication systems should be implemented on a trial basis so as to further verify their applicability.
- 2. The preliminary investigations concerning the development of a technique for predicting the reliability of electronic data processing systems, which were discussed in the main body of this report, should be continued and finalized.
- 3. It is recommended that the results of this study be expanded so as to provide means for predicting the availability of proposed systems. This would necessitate investigation and development of techniques for predicting the maintainability of such systems during the early design stages.

In the case of ground communication systems, consideration would also have to be given to the number of links (hops) in a system, the expected number of channels, propagation effects and other factors inherent in availability deter-

minations. Similar factors would likewise have to be considered for radar and electronic data processing configurations. It is believed that the development of techniques for predicting availability would be of invaluable assistance to the Air Force in the early stages of system planning.

APPENDIX I

SUMMARY OF PROCEDURES FOR RELIABILITY

PREDICTION OF RADAR AND GROUND

COMMUNICATION SYSTEMS

APPENDIX I

SUMMARY OF PROCEDURES FOR RELIABILITY PREDICTION OF RADAR AND GROUND COMMUNICATION SYSTEMS

INTRODUCTION

The text in the main body of the report presented the details associated with the development of techniques for predicting the reliability of radar and ground communication systems during the early planning stages. The purpose of this appendix is to provide step-by-step procedures for implementing these techniques. Whenever possible the techniques have been simplified as much as possible. For example, the calculation of the Noise Safety Margin (NSM) has been simplified by direct tabular and graphical reference to the db difference between initial design and worst degradation conditions allowable.

The following is the step-by-step procedures for reliability prediction of radar and ground communication systems.

RADAR SYSTEMS

PULSE RADAR TRANSMITTERS

- 1) Determine the Peak Pulse Power, in kw.
- 2) Go to Figure 2, page 15. Draw a straight line between the Peak Pulse Power and the number of Active Element Groups proposed in the design. If no decision as to the number of active element groups can be made, the number 18 is suggested as typical, and may be used as an approximation. Read the total transmitter failure rate (λ_T) on the center scale.
- 3) Record the total transmitter failure rate (λ_T) determined in Step 2 above for later use.

RADAR RECEIVERS

- 4) Determine the ideal design value of the receiver noise figure of merit in db. This may be S/N ratio, Noise Figure, or Minimum Detectable Signal.
- 5) Determine the limiting value of the above figure of merit; that is, the value beyond which the performance of the receiver is degraded beyond useability. This value is also in db.
- 6) Note the absolute value of the difference between the values found in Steps 4 and 5 above.
- 7) Take the value obtained in Step 6 and do either of the following:
 - a) Go to Figure A-I and read from the graph the failure rate per Active Element Group for receivers corresponding to this db difference, or;
 - b) Go to Table A-I, "Table of Noise Safety Margins Equivalent to Given db Differences Between Initial Noise Figure of Merit and Worst Allowable", and find the NSM corresponding to the difference in db (Δ db) noted. Insert this value of NSM into the equation:

$$\lambda_{R} = 43.6 \log_{10} (2.5/NSM)$$

and solve for failure rate per active element group.

- 8) Multiply the value derived in Step 7a or 7b by the number of active element groups in order to determine the total receiver failure rate.

 If no decision as to the number of active element groups can be made,

 87 may be assumed as typical.
- 9) Record the total receiver failure rate (λ_{R}) for later use.



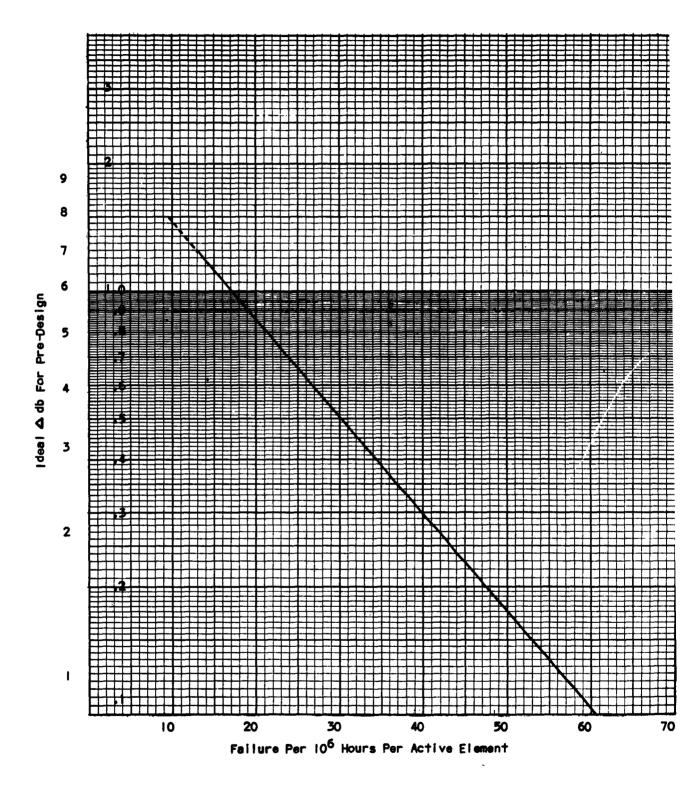


FIGURE A-I. RECEIVER NOISE SAFETY MARGIN VERSUS FAILURES PER ACTIVE ELEMENT

TABLE A-!

TABLE OF NOISE SAFETY MARGINS EQUIVALENT TO GIVEN db DIFFERENCES
BETWEEN INITIAL NOISE FIGURE OF MERIT AND WORST ALLOWABLE

△ db	<u>NSM</u>	<u> </u>	<u>NSM</u>
9.5	1.985	4.7	.718
9.4	1.951	4.6	.698
9.3	1.917	4.5	.679
9.2	1.884	4.4	.660
9.1	1.851	4.3	.641
9.0	1.818	4.2	.622
8.9	1.786	4.1	. 604
8.8	1.754	4.0	.585
8.7	1.723	3.9	.567
8.6	1.691	3.8	.549
8.5	1.661	3.7	.532
8.4	1.630	3.6	.514
8.3	1.600	3.5	.497
8.2	I . 570	3.4	.479
8.1	1.541	3.3	.462
8.0	1.512	3.2	.445
7.9	1.483	3.1	.429
7.8	1.455	3.0	.413
7.7	1.427	2.9	.397
7.6	1.390	2.8	.380
7.5	1.371	2.7	.365
7.4	1.344	2.6	.349
7.3	1.318	2.5	.334
7.2	1.291	2.4	.318
7.1	1.265	2.3	.303
7.0	1.239	2.2	.288
6.9	1.214	2.1	.274
6.8	1.188	2.0	.259
6.7	1.163	1.9	.245
6.6	1.138	1.8	.230
6.5	1.114	1.7	.216
6.4	1.089	1.6	.202
6.3	1.066	1.5	.189
6.2	1.042	1.4	.175 .162
6.1	1.019	1.3	.148
6.0	.995	1.2	.140
5.9	.973	. .0	.122
5.8	.950		.109
5.7	.928	.9 .8	.0965
5.6	•905		.0840
5.5 5.4	.884 .862	.7	.0715
5.4 5.3	.841	.5	.0593
5.3 5.2	.820	.4	.0471
5.2 5.1	.799		.0352
5.0	.778	.3 .2	.0233
4.9	.758	.1	.0117
4.8	.738	.0	.0000
4.0	. 1 20	••	

RADAR INDICATORS

PPI Scopes

- if no decision as to the number of active element groups in the proposed design. the number 30 is suggested as typical and may be used as an approximation.
- II) Multiply this by 9.96 in order to estimate the failure rate in failures per 10^6 hours.
- 12) Record this failure rate (λ_{ip}) for later use.

"A" Scopes

- 13) Determine the number of active element groups in the proposed design.

 If no decision as to the number of active element groups can be made

 the number 15 is suggested as typical and may be used as an approximation.
- 14) Multiply the value obtained in Step 13 by 11.46 in order to estimate the failure rate in failures per 10^6 hours.
- 15) Record this failure rate (λ_{1A}) for later use.

COMPLETE PULSE RADAR SYSTEMS

- 16) Sum the total failure rates (λ_T , λ_R , λ_{1P} , and λ_{1A}) obtained in Steps 3, 9, 12 and 15 above.
- 17) Divide this sum into 10^6 hours to obtain the predicted MTBF of the radar system as follows:

Radar System MTBF_R =
$$\frac{1,000,000}{\lambda_T + \lambda_R + \lambda_{1P} + \lambda_{1A}}$$



TRANSMITTER - EXCITER - RECEIVER FUNCTIONS (COMBINED)

1) Determine the system carrier-to-noise ratio from the following equation:

$$C/N = P_{+} - L_{+} + G_{+} - L_{fs} - L_{BH} + G_{R} - L_{R} + G_{FM} + D_{C} - N + B + K_{N}$$
 in db. where:

P₊ = transmitter operational power

L+ = transmission - line losses

 G_{+} = transmitting antenna gain over an isotropic path

 L_{fs} = free space loss

 $L_{\rm RH}$ = beyond the horizon loss

 G_{p} = receiving antenna gain (same as G_{+})

 L_R = receiver line losses (same as L_+)

 $G_{FM} = FM$ gain for deviation ratio (SSB=1.0)

 D_C = receiver diversity combination gain

N = receiver noise figure in db below ! watt

B = $10 \log b_{kc} + 10$ where b is the receiver bandwidth in kilocycles

 $K_N = 0.01 \text{ KT where } K = \text{Boltzmann's constant and T is } 293^{\circ}$

- 2) Go to Figure 6 on page 40 and read the estimated MTBF corresponding to this C/N ratio.
- 3) Take the reciprocal of the value obtained in Step 2 and express in failures per 10^6 hours.
- 4) Record this failure rate (λ_{TFR}) for later use.

MULTIPLEX EQUIPMENT

5) Determine the maximum number of available channels in the multiplex and divide this number by 12.

- 6) Take the quotient of the value found in Step 5 and multiply by 661.0 in order to determine the failure rate in failures per 106 hours.
- 7). Record this failure rate (λ_M) for later use.

COMPLETE COMMUNICATION SYSTEM

8) Sum the total failure rates obtained in Steps 4 and 7, $(\lambda_{TER}, \lambda_{M})$ above and divide into 10^6 hours in order to determine the MTBF of the complete Ground Communication System, as follows:

Communication System MTBF_C =
$$\frac{1,000,000}{\lambda_{\text{TFR}} + \lambda_{\text{M}}}$$